

5) PARAMETRIC EQUATIONS

The parametric equations of the 4 conic types are:

Conic type	Cartesian	Parametric
Circle	$x^2 + y^2 = r^2$	$x = r \cos \theta$ $y = r \sin \theta$
	$(x - h)^2 + (y - k)^2 = r^2$	$(x - h) = r \cos \theta$ $(y - k) = r \sin \theta$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$x = a \cos \theta$ $y = b \sin \theta$
	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	$(x - h) = a \cos \theta$ $(y - k) = b \sin \theta$
Parabola	$y^2 = 4ax$	$x = at^2$ $y = 2at$
	$(y - k)^2 = 4a(x - h)$	$(x - h) = at^2$ $(y - k) = 2at$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$x = a \sec \theta$ $y = b \tan \theta$
	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	$(x - h) = a \sec \theta$ $(y - k) = b \tan \theta$

Examples: Find the parametric equations of

1) $y^2 = 12x$

$$4a = 12, a = 3. \quad x = at^2 \Rightarrow x = 3t^2 \quad y = 2at \Rightarrow y = 6t$$

2) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

$$a = 3, b = 5. \quad x = a \sec \theta \Rightarrow x = 3 \sec \theta \quad y = b \tan \theta \Rightarrow y = 5 \tan \theta$$

3) $(x - 2)^2 + (y + 1)^2 = 4^2$

$$r = 4$$

$$(x - h) = r \cos \theta \Rightarrow (x - 2) = 4 \cos \theta \quad (y - k) = r \sin \theta \Rightarrow (y + 1) = 4 \sin \theta$$

Worksheet, Delta Ex 38.3 pg 380 Q2, 3, 5

Delta Ex 38.4 pg 381 Q2 – 5

Delta Ex 38.5 pg 382 Q2 – 5